DEFORMATION AND DISINTEGRATION OF LIQUID DROPLETS IN A GAS FLOW

M. S. Volynskii and A. S. Lipatov UDC 541.18.053

A theory of elliptic deformation and disintegration of a droplet in a gas is given for sudden application of the load. Experimental data on the determination of the critical Weber numbers for progressively increasing external forces are presented.

The laws of deformation and atomization of droplets by a gas flow are important in relation to the study of flows in the nozzles and combustion chambers of jet engines, boilers, etc.

1. The elliptic deformation theory describes the process in a constant-velocity gas [1]. It is based on the energy method and differs from other theories in that it takes into account the change in the flow over the droplet resulting from deformation. Usually (for example, in inviscid theory [2]), the pressure distribution on a sphere is employed. Let a stream of gas flow over a stationary droplet at a velocity U relative to its center of mass. We make the following basic assumptions:

1) the flow of gas and the liquid within the droplet is assumed to be inviscid and potential;

2) the deformed droplet takes the shape of an ellipsoid of revolution with minor axis directed along the gas-velocity vector.

3) evaporation of the droplet is disregarded.

The potential φ of the flow of incompressible liquid within the droplet satisfies the Laplace equation

$$\Delta\varphi = 0. \tag{1}$$

At the liquid surface with equation $F(\vartheta, \lambda, r, \tau) = 0$ the boundary conditions (Neumann problem)

$$\frac{\partial\varphi}{\partial N} = -\frac{\dfrac{\partial F}{\partial\tau}}{|\nabla F|}, \tag{2}$$

are satisfied. The energy equation is written in the integral form

$$\frac{d}{d\tau}\int\limits_{U}\frac{\rho_{\mathrm{d}}V_{\mathrm{d}}^2}{2}\,dU = \int\limits_{S} p_{\Sigma}V_N dS. \tag{3}$$

The Neumann problem is solved approximately. The velocity potential is found in the form

$$\varphi = \frac{V_c(\tau)}{2}\left(\frac{r}{r_0}\right)^2 P_2(\cos\vartheta). \tag{4}$$

Expression (4) quite closely describes the process at small deformations. However, the results of the solution can be extended (as in [2]) to large deformations.

The work done by the external forces is calculated from the usual equation for the hydrodynamic pressure distribution on an ellipsoid with semiaxis ratio $n = b/c$ (see [3]).

The expression for the work done by the pressure forces takes the form

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$$\Pi_a = 2\pi r_0^3 \rho_r u^2 \int_1^n \frac{\gamma^2}{3} \left[A(\vartheta) h^2 \frac{\partial h}{\partial n} d(\cos\vartheta) \right] dn,$$

$$\gamma = 1 + \frac{\sqrt{n^2-1} - \operatorname{arc\,tg}\sqrt{n^2-1}}{\arcsin\sqrt{1-n^{-2}} - \frac{\sqrt{n^2-1}}{n^2}}, \quad A(\vartheta) = \frac{\sin^2\vartheta}{\sin^2\vartheta + n^4\cos^2\vartheta}, \tag{5}$$

$\pi = 3{,}14$, $h = \frac{r(n, \vartheta)}{r_0}$ is the droplet surface equation.

The energy of the surface tension forces Π_σ is expressed in terms of the surface area of the ellipsoid

$$\Pi_\sigma = \sigma(S - S_0); \; S = \frac{4\pi b}{\sqrt{2}} \sqrt{b^2 + c^2}. \tag{6}$$

We introduce dimensionless time ξ in accordance with the relation

$$\tau = \xi \frac{r_0}{u} \sqrt{\frac{\rho_d}{\rho_g}}.$$

Substituting in the energy equation the expressions for Π_a and Π_σ, we obtain the equation for the deformation of the liquid ellipsoid in the gas flow

$$\frac{d^2n}{d\xi^2} = \frac{1.67}{n}\left(\frac{dn}{d\xi}\right)^2 - \frac{10.7n^2(2n^2-1)}{\mathrm{We}\sqrt{n^2+1}} - 3.75 n^{\frac{7}{3}} \gamma^2 \left\{ \frac{3}{n^2-1} + \frac{3}{(n^2-1)^2} - \operatorname{arc\,tg}\sqrt{n^2-1} \left[\frac{1}{\sqrt{n^2-1}} + \frac{4}{(n^2-1)^{3/2}} + \frac{3}{(n^2-1)^{5/2}} \right] \right\}. \tag{7}$$

In the limit as $n \to \infty$ the second term of the equation tends to $-n^3/\mathrm{We}$, and the third term to $n^{13/3}$.

Consequently, there is a deformation "energy barrier," at We > $\mathrm{We_{cr}}$, where $\mathrm{We_{cr}}$ is a characteristic constant of the process, at which the work done by the external forces tends to infinity.

The results of a numerical integration of Eq. (7) are presented in Fig. 1.

The solutions may be divided into two families: at small We the deformations are oscillatory in character, at large We they are aperiodic, as the droplet tends to an infinitely thin ellipsoid.*

The characteristic Weber number separating these families is given by the inequality $5.4055 < \mathrm{We_{cr}} < 5.408$ and may be interpreted as the critical value; it corresponds to a semiaxis ratio of 3.75. The thinning liquid disk is unstable and is broken up by small perturbations. Spark photographs of droplets at high postcritical Weber numbers show that their shape approaches that of a disk, from which the crests of waves of growing amplitude are observed to separate.

In [4] $\mathrm{We_{cr}}$ is determined from the condition of loss of static stability of a stationary liquid ellipsoid. It occurs at We = 3.75 and n = 6.

In our dynamic model $\mathrm{We_{cr}}$ is found to be greater at a smaller semiaxis ratio. The result given by the theory determines the lower limit of We numbers corresponding to large Reynolds numbers $\mathrm{Re} \geq 10^6$, where the flow is close to potential and the liquid inviscid. For these conditions the Isshiki theory [2] gives $\mathrm{We_{cr}} = 5.3$.

Figure 1 shows that as We $\to \infty$ there is a minimum dimensionless deformation time $\xi_{min} = 0.8$ (which is of the same order as the critical deformation time in experiments with inviscid droplets at various Re). For this case Isshiki obtains 1.1.

2. Our experimental procedure for investigating the atomization of a droplet in the presence of smoothly increasing external forces was as follows. The droplet generator ejected (almost without initial velocity) individual droplets of known size. At the outlet they entered a free stream of air, moving in the variable-velocity zone of its boundary layer (see Fig. 2).

* Under certain conditions the pressures p in the high-curvature zones of the ellipsoid may become negative; naturally, only physically realistic solutions at p > 0 are considered.

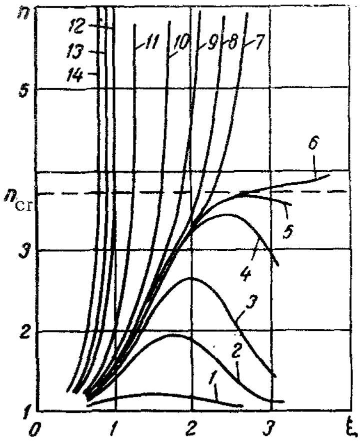

Fig. 1

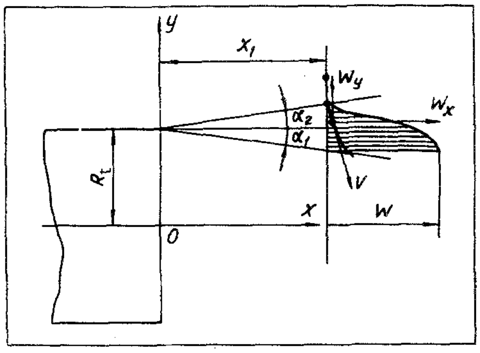

Fig. 2

Fig. 1. Development of the deformation in time at various Weber numbers: 1) We = 4; 2) 5; 3) 5.3; 4) 5.4; 5) 5.4055; 6) 5.408; 7) 5.42; 8) 5.44; 9) 5.5; 10) 6; 11) 7; 12) 10; 13) 15; 14) 30.

Fig. 2. Motion of droplet in a free jet of initial radius R_t: V is the droplet velocity; W_x and W_y are the gas velocity components; W is the gas velocity on the jet axis.

The exit velocity was increased, until atomization of the droplet was confirmed by two independent methods – by photography and by trapping on special screens (for details on the operating principle of the droplet generator and the experimental procedure see [5]).*

In order to determine the Weber number from the relative velocity of the droplet as a function of time we numerically integrated the equation of motion, in which aerodynamic and gravitational forces were taken into account. In vector form it may be written (Fig. 2)

$$m\frac{d\mathbf{V}}{d\tau}=\frac{c_x\rho_g f_m}{2}|\mathbf{W}-\mathbf{V}|(\mathbf{W}-\mathbf{V})+m\mathbf{g},$$
$$|\mathbf{g}|=9.81\ \mathrm{m/sec^2}. \tag{8}$$

The gas velocity components W_x, W_y, known functions of the complex Z, are determined from free-jet theory [6] (under our experimental conditions the jet may be considered plane)

$$Z=\frac{y-R_t+x\,\mathrm{tg}\,\alpha_1}{x\,(\mathrm{tg}\,\alpha_2+\mathrm{tg}\,\alpha_1)}.$$

In our case R_t = 75 mm; α_1 = 7°10; α_2 = 8°20; the functions W_x and W_y depend importantly on the y coordinate, but it is possible to disregard the variation of x, setting $x = x_1$ (Fig. 2), x_1 = 15 mm.

The drag coefficient of the droplet is assumed constant, although in reality, it varies. Initially (low gas velocities) the droplet is almost in free fall (its velocity on entering the jet $V_y \approx 0.4$ m/sec).

In the zone of high relative velocities, where deformation chiefly develops, the order of the Reynolds numbers Re = 300-1200, and the c_x of a sphere varies in the interval 0.4-0.6; accordingly we may take $c_x \approx 0.5$.

Allowing for deformation, when in the critical stage the droplet is almost ellipsoidal with a semiaxis ratio of 4-6, we may roughly estimate c_x as the mean of the values for a sphere and a disk $0.4 \le c_x \le 1$. We take $c_x \approx 0.7$; 0.5.

We estimate the area f_m of the mid-section of the deformed droplet as the mean of the area of the sphere f_s and the ellipsoid in the critical phase; then $f_m \approx 2f_s$. The equation of motion (8) and the relations

* We note in passing that, in the present opinion of the author, the theoretical interpretation of the experimental data of [5] is not quite correct. The entrainment of the droplet in the boundary layer is not taken into account. However, the procedure and all the results relating to atomization velocities remain valid.

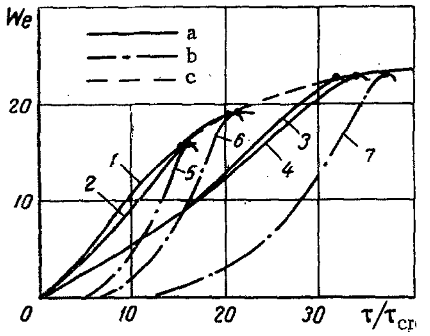

Fig. 3. Weber numbers We for a progressive increase in the external forces acting on a droplet of diameter $2r_0$ with various drag coefficients c_x: a) $c_x = 0.5$; 1) $2r_0 = 800\ \mu$; 2) 600; 3) 300; 4) 250); b) $c_x = 0.7$; 5) $2r_0 = 800\ \mu$; 6) 600; 7) 300; c) curve of critical Weber numbers We_{cr}.

for W_x and W_y reduce to three equations in V_x, V_y, and τ, unknown functions of the complex Z. After solving this system on a computer, we find the relative velocity of the droplet and the Weber number as functions of time.

The critical Weber number We_{cr} was calculated from the maximum relative velocity of the droplet in the atomization regime. This velocity is reached at the edge of the boundary layer. Atomization takes place at a Weber number close to that calculated, since at lower gas velocities disintegration of the droplet is not observed.

The possible fall of the Weber number in the core of the free jet is relatively small (less than 20%). The deformation time was found to be an order greater than the critical time τ_{cr} corresponding to a sudden application of the load. It may be assumed that the process is almost quasi-stationary, when a progressive increase in the external forces yields a small deformation increment and the inertia forces of the internal motions in the droplet can be neglected; the state of deformation is then determined by the instantaneous Weber number irrespective of the previous history of the process.

For quasi-stationary deformation the number We_{cr} is the upper limit of the atomization criterion and should exceed the critical number for a suddenly applied load.

In Fig. 3 we have plotted the theoretical dependence of the Weber number on dimensionless time τ / τ_{cr} for droplets of alcohol with different diameters $2r_0$. This ratio gives a measure of the difference between the slow process and the case of a suddenly applied load with u = W.

In accordance with the expression for τ_{cr} (see [6])

$$\frac{\tau}{\tau_{cr}} = \frac{\tau u}{\xi r_0}\sqrt{\frac{\rho g}{\rho_d}}. \tag{9}$$

The experimental data give $\xi = 3$. The dots on the curves correspond to the critical values of the Weber numbers, which lie on the dashed curve.

The process is the closer to being quasi-stationary, the smoother the variation of the Weber number, i.e., the less the gradient $dWe/d(\tau/\tau_{cr})$, which falls as the diameter decreases.

It may be assumed that the value of We_{cr} for the quasi-stationary process (it corresponds to the asymptote of the We_{cr} curve) is 22-24 at $c_x \approx 0.5$-0.7 i.e., on this interval the value of c_x has little effect on We_{cr}.

For droplets of inviscid liquids such as alcohol, water, etc., of diameter $2r_0 \geq 2000\ \mu$ the deformation on entering the free jet takes place under conditions of almost instantaneous application of the load. Apparently, for droplets of viscous liquids (for example, glycerin) the values of We_{cr} (see [2, 7]) for gradual and instantaneous application of the load are much the same owing to the viscosity effect.

After completing their investigation the authors became acquainted with the work of Gordin et al [8], which is similar in principle and technique to section 1 of the present article. Like our own work, that of Gordin et al is exceptional in taking into account the variation of the forces and interpreting the critical Weber number We_{cr} as a dynamic instability parameter. However, our theory is distinctive in the following respects: 1) the kinetic energy of the liquid in the droplet is not calculated in the same way as in [8]; 2) another method of solution, giving the behavior of the deformation in oscillatory and aperiodic regimes, is employed; 3) the minimum deformation time as We → ∞ is determined; 4) finally, our value $We_{cr} \approx 5.406$

is in agreement with the data of [2] and exceeds the static stability criterion of [4]. Gordin et al. [8] give $We_{cr} \approx 3.3$, which is less than the static stability criterion.

NOTATION

$We = 2r_0\rho U^2/\sigma$	is the Weber number;
We_{cr}	is the critical Weber number;
$Re = 2r_0 u/\nu$	is the Reynolds number;
c_x	is the drag coefficient;
φ, V_d	are the potential and velocity of the liquid in the droplet;
ρ_g, ρ_d	are the densities of the gas and the droplet;
U	is the volume of the droplet;
p, p_σ	are the static and capillary pressures;
p_Σ	is the sum of the static and capillary pressures;
σ	is the surface tension;
τ	is the time;
V_c	is the frontal-point velocity;
u	is the relative gas velocity;
r_0	is the initial radius of the droplet;
ϑ, λ, r	are the spherical coordinates in the system fixed in the center of mass of the droplet;
b, c	are the semimajor and semiminor axes of the droplet;
F, S, S_0	are the surface equation of the droplet and its area in the deformed and initial states;
P_2	is the second-order Legendre polynomial;
x, y	are the fixed coordinates;
W_x, W_y, V_x, V_y	are the components of the gas and particle velocities;
R_t	is the tube radius;
f_m	is the area of the mid-section of the droplet;
m	is the particle mass;
W	is the gas velocity on the jet axis;
ν	is the kinematic viscosity of the gas;
N	is the normal to the droplet surface;
W, **V**	are the velocities of the gas and the center of mass of the droplet;
g	is the acceleration of gravity.

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